Course 10  Rational Expressions

Module:  Evaluating Rational Expressions

Objective:  To practice evaluating a rational expression for a given set of values

Name:  _______________________  Date:  __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ Any number that can be written as a ratio of two integers is a ________________ (rational, valid) number.

➢ Can polynomials be classified as rational expressions if they are written in the form \( \frac{p}{q} \) where \( q = 1 \)? ________________ (yes or no)

➢ The set of all fractional numbers is sometimes called the set of ________________ ________________.

➢ How can you represent 2, \( x + 2 \), and .4 as rational expressions?

__________________ __________________ __________________

Problem Set:
Find the value of these expressions for the specified replacements of \( a, b, \) and \( c \).

\[ a = 2 \quad b = -3 \quad c = 1 \]

\[ 3a \quad 5b - 3 \quad 4b^2 \]

\[ 3a^2 - b^3 \quad 4b^2 \quad \frac{a + b + c}{2} \]
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Evaluate these expressions for \( a = 2 \) and \( c = 3 \). Show your work.

\[
2(a + c) - \frac{3}{c - a} + 7 = \text{__________________________}
\]

\[
\frac{9}{c} - \frac{a}{2} + \frac{5ac}{a + c} = \text{__________________________}
\]

\[
\frac{2ac}{4} + \frac{1}{2} (4c - a) = \text{__________________________}
\]

\[
\frac{a^2 c}{4} + \frac{ac^2}{6} + c = \text{__________________________}
\]

Reflection:
Explain the rules for rational expressions and for rational numbers as if you were the teacher describing them to a student for the first time.

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Course 10  Rational Expressions

Module: Restrictions on Rational Expressions

Objective: To practice identifying nonpermissible values for the variables in a rational expression

Fill in the blanks. Determine whether each statement below is true or false. Write the correct response in the blank.

- In a rational expression, any number may serve as the denominator. __________
- When a denominator is zero, the expression is undefined. __________
- In a rational expression, the variables may not be replaced by numbers that make the denominator equal to 0. __________
- In some expressions you must solve the quadratic equation in the denominator to find nonpermissible replacements for variables. __________

Problem Set:
Find the nonpermissible replacement for the variables in these expressions.

\[
\frac{3x}{5x} \quad \frac{y + 3}{4x - 1} \\
\frac{5}{2x - 2} \quad \frac{2}{(x - 3)^2}
\]
Extension Activity:

Evaluate \( \frac{x^2 + 4x + 3}{x^2 + 8x + 15} \) and show your work when:

1. \( x = 3 \)
2. \( x = -3 \)
3. \( x = 0 \)

Reflection:

Explain how to identify nonpermissible values for the variables in a rational expression. Use this expression to find the nonpermissible replacement for \( x \) in your explanation:

\( \frac{x^2}{2x + 6} \)
Course 10       Rational Expressions

Module: Equivalent Forms of Rational Expressions

Objective: To practice identifying rational expressions that are equivalent

Name: _______________________ Date: __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ When two fractions are both representations of the same number, they are _________________ fractions. (equivalent, nonequivalent)

➢ The rule states that if you multiply or divide the numerator and denominator by the _________________ number, you always get an _________________ fraction.

➢ Cross multiplying works with rational expressions. _________________ (true, false)

➢ You can always check to see whether expressions are equivalent by choosing replacements for the variables in each expression. _________________ (true, false)

Problem Set:
In each problem, tell whether the rational expressions are equivalent or nonequivalent.

\[
\frac{2x^2}{4} \quad \frac{x^2}{2} \quad \text{__________________________}
\]

\[
\frac{ab^2}{ab} \quad \frac{ab^3}{ab^2} \quad \text{__________________________}
\]

\[
\frac{b + 2}{b^2 + 2} \quad \frac{1}{b} \quad \text{__________________________}
\]
Reflection:
Explain the rule about equivalent fractions in your own words. Use an example as part of your explanation.

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\[
\frac{3rst}{15rt} \quad \frac{s}{5} \\
\frac{x + 1}{x^2 - 1} \quad \frac{x}{x - 1} \\
\frac{2x + 4}{4} \quad 2 \\
\frac{x^2 y^3 z}{xy^2} \quad \frac{xz}{y^{-1}} \\
\frac{x - 9}{x + 3} \quad x - 3 \\
\frac{10y^3}{y} \quad 5y^2 \\
\frac{7}{14xyz} \quad \frac{1}{2}xyz \\
\frac{4}{s^2} \quad \frac{4(t + 1)}{s^2 t + s^2}
\]
Course 10  Rational Expressions

Module:  Simplifying Rational Expressions

Objective:  To practice simplifying rational expressions

Name:  _______________________  Date:  __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ The first step in simplifying a rational expression is to factor the __________________ and the __________________.

➢ You cannot cancel terms found within parentheses when simplifying an expression. __________________ (true, false)

➢ To simplify an expression you must cancel out the common factor. __________________
  (true, false)

➢ All rational expressions can be simplified. __________________
  (true, false)

Problem Set:
Simplify these expressions.

1.  \[ \frac{12ab^2}{8ab} \]  2.  \[ \frac{2(x - y)}{2(x + y)} \]

3.  \[ \frac{14x^2y^2z}{-2xz} \]  4.  \[ \frac{(x + y)(x + 3)}{(x + 3)(x - y)} \]

5.  \[ \frac{b - 3}{b^2 - 6b + 9} \]  6.  \[ \frac{5x - 5}{x^2 - 1} \]
# Course 10  
## Rational Expressions

### Problem Set

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| 7. | \[
\frac{3(a - b)}{3a - 3b}
\] |   |
| 8. | \[
\frac{3x^2y}{6x}
\] |   |
| 9. | \[
\frac{5s^7}{10s^2t}
\] |   |
| 10. | \[
\frac{t^2 - 4}{t + 2}
\] |   |
| 11. | \[
\frac{b^2(a + 2)}{b(a + 2)^2}
\] |   |
| 12. | \[
\frac{x^2 + 7x + 10}{x^2 - x - 6}
\] |   |
| 13. | \[
\frac{3xy - 6y}{4x^2y - 8xy}
\] |   |
| 14. | \[
\frac{6m + 18}{2m^2 + mn + 6m + 3n}
\] |   |

### Reflection:

Explain the steps used to simplify rational expressions. Use this expression to illustrate the steps.

\[
\frac{x^2 - 4}{x^2 + x - 6}
\]
Module: Sum of Rational Expressions, Part 1

Objective: To practice finding the sum of rational expressions with like denominators

Name: _______________________ Date: __________________

Fill in the blanks. Determine whether each statement below is true or false. Write the correct answer in the blank.

- The rules for adding rational expressions are the same as those for adding fractions. _________________
- The rule for adding fractions tells us to add the terms in the numerator. _________________
- When adding fractions with common denominators, the denominator does not change. _________________

Problem Set:
Express each sum in the simplest form.

1. \( \frac{3}{a} + \frac{5}{a} \) _________________
2. \( \frac{x}{x + 3} + \frac{3}{x + 3} \) _________________

3. \( \frac{-9}{t - 3} + \frac{t^2}{t - 3} \) _________________
4. \( \frac{3y}{2y + 1} + \frac{6y}{2y + 1} \) _________________

5. \( \frac{b}{b - 5} + \frac{2b}{b - 5} \) _________________
6. \( \frac{a}{a + 3} + \frac{-2a + 3}{a + 3} \) _________________
7. \[ \frac{s^2}{s+3} + \frac{6s+9}{s+3} \]

8. \[ \frac{x-3}{x^2-9} + \frac{x-3}{x^2-9} \]

9. \[ \frac{-b+2}{b-3} + \frac{2b-5}{b-3} \]

10. \[ \frac{2}{3y+3} + \frac{1}{3y+3} \]

11. \[ \frac{-5}{3s-5} + \frac{3s+1}{3s-5} \]

Reflection:
Explain the rule for adding a rational expression with the same denominator. Explain this as if you were the teacher explaining it to a student for the first time. Use an expression to illustrate the rule.

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Course 10  Rational Expressions

Module: Difference of Rational Expressions, Part 1

Objective: To practice subtracting rational expressions with the same denominator

Name: _______________________  Date: __________________

Fill in the blanks. Determine whether each statement below is true or false. Write the correct answer in the blank.

➢ The rule for subtracting rational expressions is different from the rule for subtracting fractions. _____________________

➢ To find the difference of fractions with the same denominator, subtract the numerators. ______________________

➢ You always combine like terms when subtracting rational expressions. ______________________

Problem Set:
Express each difference in simplest form.

1. \( \frac{5}{b} - \frac{2}{b} \)  \[ \]  
2. \( \frac{b}{b - 2} - \frac{4b + 2}{b - 2} \)  \[ \]

3. \( -\frac{x}{x + 3} - \frac{2}{x + 3} \)  \[ \]  
4. \( \frac{x - 2}{x - 1} - \frac{x^2 - 2}{x - 1} \)  \[ \]

5. \( \frac{4}{x + 1} - \frac{2x + 2}{x + 1} \)  \[ \]  
6. \( \frac{b^2}{b + 5} - \frac{25}{b + 5} \)  \[ \]
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7. \( \frac{3t}{2t+1} - \frac{6t}{2t+1} \)

8. \( \frac{xyz}{xy-1} - \frac{z}{xy-1} \)

9. \( \frac{2r}{2r-s} - \frac{r}{2r-s} \)

10. \( \frac{x-3}{x^2-3} - \frac{x-3}{x^2-3} \)

11. \( \frac{s^2}{s-3} - \frac{3s-2}{s-3} \)

12. \( \frac{x}{3y+3} - \frac{4x}{3y+3} \)

Extension Activity:
Explain the rule for subtracting fractions that have the same denominator.
Illustrate the rule showing a pie shape.
Fill in the blanks.

- The rules for multiplying rational expressions are the same as those for multiplying ___________ ___________ or ___________.

- What are the three steps for multiplying rational expressions?
  1. Multiply the ________________.
  2. Multiply the ________________.
  3. ________________ the expression.

- If the numerator and the denominator have a ________________ ________________ ________________, the rational expression can and should be simplified.

Problem Set:
Express each product in simplest form.

1. \( \frac{6}{y} \cdot \frac{3y}{6} \) ________________________________

2. \( \frac{3x}{4y} \cdot \frac{y^2}{3x} \) ________________________________

3. \( \frac{a-b}{a+b} \cdot \frac{a+b}{a-b} \) ________________________________

4. \( \frac{xy}{2z} \cdot \frac{xy}{4z} \) ________________________________

5. \( \frac{x^2 - y^2}{12x^2y} \cdot \frac{6x}{x-y} \) ________________________________
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6. \( \frac{r + 2}{3r^3} \cdot \frac{6r}{r + 2} \)  

7. \( \frac{3x - 6}{x + 3} \cdot \frac{2x + 6}{x - 2} \)  

8. \( \frac{4y^2z}{x^2 + 2x} \cdot \frac{2x^2}{8y^2z} \)  

9. \( \frac{15x^2}{2x + 4} \cdot \frac{x^2 - 4}{5} \)  

10. \( \frac{2y + 10}{x} \cdot \frac{xy}{y^2 + 6y + 5} \)  

Extension Activity:  
Express the product in simplest form.

\( \frac{48a^2br^3x^2 + 24a^2b^3r^3xy}{96abz^2} \cdot \frac{72a^2x^3yz^4}{16ar^2x^2y^3 + 8ab^2r^2xy^4} \)  

Reflection:  
Do you prefer to simplify before or after multiplying?

________________________________________________________________________

Explain why.

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Module: Quotient of Rational Expressions

Objective: To practice finding the quotient of two rational expressions

Name: _______________________  Date: __________________

Fill in the blanks.

➢ The rules for dividing rational expressions are the same as those for dividing ________________.

➢ What are the three steps for finding the quotient of two rational expressions?

   1. ________________ the divisor and multiply.

   2. Multiply the ________________ to get the numerator of the solution. Then, ________________ the denominators to get the denominator of the solution.

   3. Check to see if the expression needs to be ________________.

➢ Finish the rule: \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \)

Problem Set:
Express each quotient in simplest form.

1. \( \frac{5}{y} \div \frac{10}{y} \)  ________________

2. \( \frac{a}{4b} \div \frac{b^2}{5a^2} \)  ________________

3. \( \frac{r - 5}{r + 5} \div \frac{r - 5}{r + 5} \)  ________________
Course 10

Rational Expressions

4. \(\frac{6xy}{z} \div \frac{4x^2y}{xyz}\)

5. \(\frac{x+y}{x-y} \div \frac{3x^2-3y^2}{3y-3x}\)

6. \(\frac{r+2}{r^2s} \div \frac{r+2}{rs^2}\)

7. \(\frac{5x-10}{x+3} \div \frac{3x-6}{2x+6}\)

8. \(\frac{b+3}{2b} \div \frac{b^2-9}{b}\)

9. \(\frac{7t}{t+2} \div \frac{7t+14}{t}\)

10. \(\frac{y^2+y}{y} \div (y+1)\)

Extension Activity:
Circle the expressions that are in simplest form. If the expression is not in simplest form, simplify it on the space provided.

\(\frac{7x}{y}\) \(\frac{x}{2}\) \(\frac{z^2}{z}\)

\(\frac{4ab}{9p}\) \(\frac{3x}{21y}\) \(\frac{4r^2s}{12s^2}\)

Reflection:
What process do you follow for finding the quotient of two rational expressions? Work through an example to demonstrate these steps.

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Module: Common Denominators of Rational Expressions

Objective: To practice finding the least common denominator of two rational expressions

Name: _______________________  Date: __________________

Fill in the blanks.

Finding the least common denominator of two rational expressions is similar to finding the least common denominator of two ____________.

What are the steps for finding the least common denominator?

1. Factor each _______________ completely.

2. Multiply all the different _______________ _______________.

   For each factor, use the greatest exponent.

To find the sum or difference of two rational expressions with unlike denominators, it is often helpful to use the _______________ _______________._______________.

Problem Set:
Find the least common denominator for each pair of rational expressions.

1. \( \frac{5}{12x} + \frac{x}{3} \)
2. \( \frac{1}{x^2 - y^2} + \frac{2}{x + y} \)
3. \( \frac{1}{3ab^2} + \frac{5}{9b^2c} \)
Rational Expressions

4. \( \frac{x}{7x^2 - 7y^2} \) \( \frac{x^2}{x - y} \)

5. \( \frac{-3}{x^2 - 4x + 3} \) \( \frac{5}{x - 1} \)

6. \( \frac{x}{8x^2 - 18} \) \( \frac{2x}{2x + 3} \)

7. \( \frac{x}{x - 5} \) \( \frac{3}{x^2 - 3x - 10} \)

8. \( \frac{4x}{x^2 - 1} \) \( \frac{x^2}{x^2 + 4x + 3} \)

9. \( \frac{x}{x + 1} \) \( \frac{x + 1}{x - 1} \)

10. \( \frac{n + 4}{4} \) \( \frac{n}{4n - 8} \)

Extension Activity:
Draw a line from each pair of expressions on the left to its least common denominator on the right.

\( \frac{2x}{x^2 - 6x + 9} \) \( \frac{x^2}{x^2 - x - 6} \) \( (x - 3)(x + 3)(x - 2) \)

\( \frac{3}{x^2 - 9} \) \( \frac{x}{x^2 + x - 6} \) \( (x - 2)(x + 4)(x - 3) \)

\( \frac{2x}{x^2 + 6x + 8} \) \( \frac{4x}{x^2 + 3x - 4} \) \( (x + 2)(x - 3)^2 \)

\( \frac{3x^2}{x^2 + 2x - 8} \) \( \frac{4}{x^2 + x - 12} \) \( (x + 2)(x + 4)(x - 1) \)

Reflection:
Explain the process you use to find the least common denominator.
Fill in the blanks. Use one of the words in parentheses when choices are given.

- The first step in finding the sum of two rational expressions is to find the
  _______________________________ _______________________________ _______________________________.

- This sum is in the simplest form \( \frac{5m + 3n}{m(m + n)} \) __________________ (true, false)

- In the example \( \frac{8x}{x^2 + 4x + 4} + \frac{5}{x^2 + 3x + 2} \) what are the prime factors?
  _______________________________ _______________________________

- In the example \( \frac{1}{b^2c} + \frac{b}{c^2} \) what is the least common denominator?
  _______________________________

Problem Set:
Find the sum. Show your work.

1. \( \frac{3}{x^2y^4} + \frac{2}{x^3y^2} \)

2. \( \frac{2x}{x^2 - 9} + \frac{2}{x + 3} \)
Rational Expressions

3. \( \frac{n+2}{3} + \frac{n}{3n-6} \)

4. \( \frac{r}{6st} + \frac{5}{3rst} \)

Reflection:
Explain how to find the sum of two rational expressions with unlike denominators as if you were a teacher explaining it to a student for the first time.

Use this problem in your explanation: \( \frac{2}{m+n} + \frac{3}{m} \)
Module: Difference of Rational Expressions, Part 2

Objective: To practice finding the difference of rational expressions with different denominators

Name: _______________________ Date: ________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

- When you form a product of all the different prime factors, for each factor use its __________________ exponent. (smallest, greatest)
- When finding the difference of rational expressions with unlike denominators, be sure your answer is in ________________ terms.
- In the problem $\frac{3}{x^2 - 4x} - \frac{2}{x^2 - 16}$ what is the least common denominator?

Problem Set:
Find the difference. Show your work.

1. $\frac{b}{ac^2} - \frac{c}{ab^2} = ________________________$

2. $\frac{x}{x - y} - \frac{3}{x^2 yz} = ________________________$
Course 10  
Rational Expressions

\[ \frac{3}{a} - \frac{2+b}{ab} = \quad \text{________________________} \]

4. \[ \frac{2}{y} - \frac{y+2}{3y} = \quad \text{________________________} \]

5. \[ \frac{7}{x^2 + 6x} - \frac{5}{x^2 + 12} = \quad \text{________________________} \]

Reflection:
What are the easiest and hardest things to remember when finding the difference of rational expressions with unlike denominators?
Explain using this problem: \[ \frac{3}{x^2 - 4x} - \frac{2}{x^2 - 16} \]

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Course 10  
Rational Expressions

Module:  Review: Rational Expressions

Objective:  To review rational expressions

Name:  _______________________  Date:  __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ If the numerator and the denominator have a _______________ _______________, the rational expressions can and should be simplified.

➢ To find the sum or difference of two rational expressions with unlike denominators, it is often helpful to use the _______________ _______________.

➢ When you form a product of all the different prime factors, for each factor use its _______________ exponent. (smallest, greatest)

➢ When adding or subtracting fractions with common denominators, the denominator does not change. _______________ (true, false)

➢ Finish the rule: \( \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \star - \)

➢ This expression is in simplest form. \( \frac{5m + 3n}{m(m + n)} \) _______ (true, false)

Problem Set:
Find the nonpermissible replacement for the variables in these expressions.

1. \( \frac{38x}{(2x-8)^2} \)  
2. \( \frac{a+125}{a-15} \)  
3. \( \frac{14z}{41x} \)

Tell whether the rational expressions are equivalent or nonequivalent.

4. \( \frac{b^3}{b} \star \frac{x^3}{x} \)  
5. \( \frac{3c^{21}}{12c^{19}} \star \frac{c^3}{4c} \)
Course 10

Rational Expressions

Express each sum in simplest form.

6. \( \frac{n}{(m-12)^2} + \frac{nop-3n}{(m-12)^2} = \)

7. \( \frac{35ab}{6} + \frac{7ab+6}{6} = \)

8. \( \frac{r^2}{r^2+13r+42} + \frac{r-6}{r^2+12r+36} = \)

9. \( \frac{2rv}{tuv} + \frac{stv}{qrs} = \)

Express the difference in simplest form.

10. \( \frac{12-6s}{s+6} - \frac{-7s+6}{s+6} = \)

11. \( \frac{v+3}{v^2-9} - \frac{v-3}{v^2-9} = \)

12. \( \frac{b^2-b-20}{b^2-25} - \frac{1}{b-5} = \)

13. \( \frac{4}{a^2-b^2} - \frac{c}{12} = \)

Express the product in simplest form.

14. \( \frac{12s}{p+3qr^2} \cdot \frac{12s}{s} = \)

15. \( \frac{8}{12x+3} \cdot \frac{3x}{2x-4} = \)

Express each quotient in simplest form.

16. \( \frac{t-2}{r-6} \div \frac{r-6}{t-2} = \)

17. \( \frac{4}{5s-10} \div \frac{2}{5s-10} = \)
Module: The Coordinate Plane

Objective: To practice identifying the coordinates of a point on a two-dimensional coordinate graph

Name: _______________________  Date: __________________

Fill in the blanks. Use one of the terms in parentheses when choices are given.

- A one-dimensional graph used to visualize a problem with only one variable is called a _______________ _______________. (number line, simple graph)
- When a problem has two variables, the two-dimensional graph used to visualize data is called a _______________ _______________. (Cartesian coordinate, complex system) plane.
- When a graph consists of two number lines, the horizontal line is the _______________ axis and the vertical line is the _______________ axis.
- The point where the axes cross is called the _______________. (intersection, origin, quadrant)

Problem Set: Label the four quadrants in the graph.
**Course 11**

**Graphs and Linear Equations**

Name the coordinates corresponding to each point.

![Graphs](image)

**Reflection:**
History tells us that René Descartes thought of the Cartesian coordinate plane when he was lying in bed watching a fly crawl on the ceiling. He asked himself if there was a way to describe the fly’s path by noting its distance from where the edges of the ceiling met the walls.

Pretend this coordinate plane is an aerial view of your neighborhood. Place your home at the origin and locate three familiar places: school, stores, a friend’s house, and so on. The plane can represent miles or city blocks. Label each location and write its ordered pair.
Fill in the blanks. Select one of the words in parentheses when choices are given.

- Every point on a coordinate plane can be identified by an ordered pair, ordered system of numbers that determine its location.
- The first number in an ordered pair is the x-coordinate. The second number is the y-coordinate.
- The x-coordinate shows the horizontal position. The y-coordinate shows the vertical position.

Problem Set:
Plot the ordered pairs on the coordinate plane. Label each pair this way: (x,y).

(4,2)  
(5,7)  
(-6,3) 
(2,-2) 
(-2,-4) 
(3,-6) 
(-5,-7)
Extension Activity:
On the coordinate plane, plot solutions to the equation $x + y = 5$. The $x$-coordinate is provided. Label each ordered pair. When you are finished, draw a line through the points on the coordinate plane. The first point is plotted for you.

Reflection:
Suppose a friend has trouble remembering which is the $x$-axis on a coordinate plane and which is the $y$-axis. How would you help your friend remember which is which?

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Module: Solutions of Linear Equations as Ordered Pairs

Objective: To practice describing solutions to linear equations as ordered pairs

Name: _______________________ Date: ________________

Fill in the blanks. Use one of the words or phrases in parentheses when choices are given.

- The equation $2x + y = 6$ is a _________ equation.
- In a linear equation the exponent for each variable is ________.
- Every solution of a linear equation is an ________ ________ of numbers, one for $x$ and one for $y$.
- The ordered pair $(0, 6)$ ________ (is, is not) a solution to the equation $2x + y = 6$.
- To find solutions of an equation with two variables, we can__________ for one variable and ________ a value for the other variable.
- When you graph an equation, the line that you graph represents all the possible ________ of the equation.

Problem Set:
Find two ordered pairs that satisfy each of the following equations. Show your work.

1. $2x = y + 4$
2. $5x = y$
3. $y = x - 5$
4. $3x - 2 = y$
Using substitution, complete the table of solutions for each equation.

5. \(2y - 3x = 4\)  
6. \(5x + \frac{1}{2}y = 10\)

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</table>

Reflection:
In this tutorial, you learned about solutions to a linear equation as ordered pairs. Explain in your own words how you determine if an equation is satisfied or not satisfied by an ordered pair. Show an example of each.

Example 1:

Example 2:
Module: Graphing a Linear Equation in 2 Variables

Objective: To practice graphing equations in two variables on a coordinate plane

Name: _______________________  Date: ________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ When finding the solution set for an equality, if you ______________ for y, then you can ______________ a value for the variable x. (solve, substitute, inspect)

➢ When graphing the set of solutions for an equation, if a point does not fall on the line, then that ordered pair ______________ a solution of the equation. (is, is not)

➢ When you have two sets of solutions, you can graph a linear equation. ______________ (true, false)

➢ When you plot the points of the set of solutions on a graph, they fall along a straight line. ______________ (true, false)

Problem Set:
Find at least two ordered pairs that satisfy each equation, and then draw the line that is the graph of the equation.

\[2x + y = -4\] \[x + y = 7\]
Reflection:
What are the key things to remember when finding the set of solutions for an equality and graphing the solutions? How would you explain the process to a friend?

________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
Module: Graphing a Linear Inequality in 2 Variables

Objective: To practice graphing inequalities in two variables on a coordinate plane

Name: _______________________  Date: __________________

Fill in the blanks. Use one of the terms in parentheses when choices are given.

➢ To solve the inequality $y - x < 4$ for $y$, you need to isolate the variable $y$. _________ (true, false)

➢ To graph the inequality, you need to find the ______________ to establish the boundary for the inequality. (additive inverse, related equality, arbitrary points, square root)

➢ If an arbitrary point above the line makes a true statement, then all points in that half plane will satisfy the equation. (true, false)

➢ When graphing an inequality, if points along the line are included in the solution set, then the line is ______________. If points on the line are not included, use a ______________ line. (dotted, dashed, solid)

Problem Set:
Graph the inequality.

1. $y + 5x \geq 8$
2. $2y + 2x < 10$

---

**Extension Activity:**
Here’s a problem that’s a little more challenging.

$-3y \leq -6x + 12$

---

**Reflection:**
In this tutorial, you learned about graphing inequalities in two variables. In your own words, describe the steps used in this process.
Module: Slope of a Line from 2 Points

Objective: To practice finding the slope of a line given two points on the line

Name: _______________________  Date: ________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ The steepness measurement of a line is called the ________________ (angle, slope, rise).

➢ The above measurement is the ratio of how much the line ____________ to how much it runs ________________ (rises, angles, vertically, horizontally).

➢ The ________________ of a line is the difference of the y-coordinates ____________ by the difference of the x-coordinates (angle, slope, divided, multiplied).

➢ Lines that rise to the right have a ________________ slope, and lines that rise to the left have a ________________ slope. (defined, negative, positive)

Problem Set:
Determine the slope of the line that contains each pair of points.

1. (-4,1) and (-1,2)     2. (2,-3) and (0,1)
3. (3,4) and (-1,-1)     4. (2,-1) and (-3,1)
5. (7,3) and (7,-6)     6. (-2,6) and (2,3)
Graphs and Linear Equations

7. (6,0) and (0,-4)  
8. (-2,-3) and (4,-3)

9. (-3,-3) and (9,10)  
10. (-8,4) and (-3,-7)

Determine the slope of each of the following graphed lines.

\[
\text{slope } = \frac{y_2 - y_1}{x_2 - x_1}
\]

- Graph 1: \(y = \frac{1}{2}x + 1\)  
- Graph 2: \(y = -\frac{1}{3}x - 2\)  
- Graph 3: \(y = \frac{2}{3}x + 3\)  
- Graph 4: \(y = \frac{3}{4}x - 1\)

Extension Activity:

If the slope of a line is \(\frac{4}{3}\) and it passes through points (-6,-2) and (x,3), find x.

Reflection:

Explain what slope is, and how it is calculated.
Module: The \( y \)-Intercept of a Line

Objective: To practice finding the \( y \)-intercept of a line

Name: _____________________  Date: ________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

- The slope is one defining characteristic of a line. Another is the line’s \( \underline{\text{______________}} \). (\( y \)-intercept, length, coefficients)

- The \( y \)-intercept is the coordinate of the point where a line crosses the \( \underline{\text{______________}} \). (\( y \)-axis, \( x \)-axis, origin)

- All \( y \)-intercepts have an \( x \)-coordinate of \( \underline{0} \). (\( y \), 0, 1)

- All straight lines, except \( \underline{\text{______________}} \) (parallel, vertical, horizontal) ones, have a \( y \)-intercept.

Problem Set:
Find the \( y \)-intercept of the line shown on the graph. Write your answer as a coordinate pair.

\[ (__, __) \]
Reflection:
Write a definition of y-intercept.

Why don’t vertical lines have y-intercepts?
Course 11  Graphs and Linear Equations

Module: Using the Slope and y-Intercept to Graph a Line

Objective: To practice finding points on a line using the y-intercept and slope

Name: _______________________  Date: __________________

Fill in the blanks using one of the words in the parentheses.

➢ To find points on a line with a positive slope, move either ____________

(down, up) and to the ____________ (left, right) or ____________

(down, up) and to the ____________. (left, right)

➢ The y-intercept of a line is the point at which the line crosses the

________________. (x-axis, y-axis)

Problem Set:
For each graph:
1) Plot the y-intercept.
2) Use the slope to find a second point on the line.
3) Connect the points to draw the equation of the line.

y-intercept = 1; slope = 3  y-intercept = -1; slope = -4

1.  

2.  

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Extension Activity:
The graph shows the y-intercept and a second point plotted from the slope. The second point is down below the slope. Yet the slope is positive. Explain how this is possible using the definition of slope.
Module: Finding the Slope and \(y\)-Intercept from an Equation

Objective: To practice identifying the slope and \(y\)-intercept of a line from an equation in the form \(y = mx + b\)

Name: _______________________  Date: __________________

Fill in the blanks. Select one of the letters in parentheses when choices are given.

- \(y = mx + b\) is called the \(\underline{\text{_______}}\)-intercept form of a linear equation.
- The equation \(y = mx + b\) plots as a straight \(\underline{\text{_______}}\).
- \(\underline{\text{_______}}\) is the slope. \((y, m, x, b)\)
- The slope is always the coefficient of the variable \(\underline{\text{_______}}\). \((y, m, x, b)\)
- \(\underline{\text{_______}}\) is the \(y\)-intercept. \((y, m, x, b)\)
- The \(y\)-intercept is the point at which the line crosses the \(\underline{\text{_______}}\)-axis.
- In the equation \(y = 2x - 1\), the slope is \(\underline{\text{_______}}\).
- A minimum of \(\underline{\text{_______}}\) points are needed to plot a straight line.

Problem Set: Identify the slope and \(y\)-intercept for the following equations.

1. \(y = 5x + 8\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

2. \(y = -2x + 1\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

3. \(y = -9x - 2\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

4. \(y = 7x + 7\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

5. \(y = \frac{1}{3}x - 4\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

6. \(y = -\frac{3}{2}x + 2\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

7. \(y = x - 5\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)

8. \(y = -10x\)  
   \(m = \underline{\text{_______}}\)  
   \(b = \underline{\text{_______}}\)
Course 11  Graphs and Linear Equations

9.  \( y = \frac{2}{3}x \)  
10.  \( y = -4x - 20 \)  
11.  \( y = 1x + 6 \)  
12.  \( y = -x \)

\[ m = \underline{\hspace{2cm}} \quad m = \underline{\hspace{2cm}} \quad m = \underline{\hspace{2cm}} \quad m = \underline{\hspace{2cm}} \]

\[ b = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}} \quad b = \underline{\hspace{2cm}} \]

Identify the \( y \)-intercept from the graph.

13.  \( b = \underline{\hspace{2cm}} \)

14.  \( b = \underline{\hspace{2cm}} \)

15.  \( b = \underline{\hspace{2cm}} \)

16.  \( b = \underline{\hspace{2cm}} \)

Extension Activity:
Circle all the ordered pairs (points) that satisfy the linear equation. 
(Hint: Graphing the line first may save some time.)

\[ y = -2x + 1 \]

\[ (3,-5) 	(6,2) 	(0,0) 	(2,-3) 	(3,7) 	(-6,-7) 	(9,9) 	(-2,5) \]

\[ (4,-1) 	(-3,7) 	(1,0) 	(-1,3) 	(-2,2) 	(7,-4) 	(6,3) \]

\[ (2,2) 	(1,-1) 	(1,1) 	(3,-7) 	(4,4) 	(-3,-5) 	(0,1) 	(5,-3) \]
Module: Writing Equations in Slope-Intercept Form

Objective: To practice rewriting an equation in slope-intercept form

Name: _______________________  Date: __________________

Fill in the blanks.

➢ The slope-intercept form of an equation is \( y = \) _____ + ____.

➢ If an equation is not in slope-intercept form, you can rewrite the equation as an equivalent expression in slope-intercept form.

______________ (true, false)

➢ When you rewrite a linear equation you need to perform the same operation on both sides of the equation.

______________ (true, false)

Problem Set:
Rewrite each equation in slope-intercept form. Reduce fractions to lowest terms.

1. \( 4x - 2y - 12 = 0 \)  
2. \( 21 = 5x + 7y \)

3. \( 3y - 9 = 15x \)  
4. \( -8x + 5y = 0 \)
Course 11  \hspace{1cm} \textbf{Graphs and Linear Equations}

5. \( \frac{1}{4} y + x = 8 \)

6. \( 0 = \frac{1}{5} y - \frac{1}{10} x \)

7. \( 3x + 7y - 7 = 14 \)

8. \( 12 - \frac{3}{2} y + \frac{1}{2} x = 0 \)

9. \( \frac{1}{2} = \frac{1}{6} x + \frac{1}{3} y \)

10. \( 16 + 2y + 4x = 8 \)

\textbf{Extension Activity:}
Imagine that you're going on a long bike ride. Let \( y \) represent your horizontal distance, in kilometers. Let \( x \) represent your vertical distance (km). Which of these equations describes the easiest ride?

A. \( 4y - x - 8 = 0 \)
B. \( 8x - 4y = -8 \)
C. \( -14 + 7y - 7x = 0 \)

Explain why your answer is correct and illustrate your explanation on the graph.
Module: Identifying Graphs from Their Equations

Objective: To practice identifying a graph from its linear equation

Fill in the blanks. Use one of the words in parentheses for your answer.

- Using the slope-intercept form, you can determine the graph of a linear equation by identifying the _____________ (y-intercept, x-intercept) and the _____________ (slope, direction, angle).

- Is the y-intercept always enough to determine which line is the graph of an equation? ________________ (yes, no)

- Lines with positive slopes rise to the ________________ (left, right).

Problem Set:
Describe the similarities and differences of each pair of linear equations without graphing.

1. \( y = \frac{3}{4}x + 2 \)
   \( y = -\frac{2}{3}x + 2 \)

2. \( y = 4x - 3 \)
   \( y = 5x + 1 \)
Course 11  Graphs and Linear Equations

Match the following equations with the respective line on the graph.

1. \( y = \frac{-2}{3} - 2 \)  
   Belongs to line:  

2. \( y = \frac{-1}{6} + 1 \)  
   Belongs to line:

3. \( y = \frac{7}{3} x - 2 \)  
   Belongs to line:

4. \( y = \frac{-5}{2} x + 1 \)  
   Belongs to line:

5. \( y = \frac{-2}{3} + 1 \)  
   Belongs to line:

6. \( y = -3 \)  
   Belongs to line:

7. \( y = \frac{3}{4} x - 2 \)  
   Belongs to line:

8. \( y = \frac{3}{4} x + 1 \)  
   Belongs to line:

Reflection:
Describe how to distinguish the difference between equations for vertical and horizontal lines.

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Module: Parallel Lines and Their Slopes
Objective: To practice finding the slope of lines that are parallel

Name: _______________________  Date: __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ When lines are _______________ (angled, parallel, perpendicular)
  the perpendicular distance between them is constant.

➢ If a line on a graph rises three units for every five that it moves to the right,
  the slope of the line is written as ____________________.

➢ When a line rises to the left, you know it has a ______________
  (negative, positive) slope.

➢ If two different lines have the same slope, they are _______________
  (parallel, perpendicular)

Problem Set:
Determine whether the following pairs of lines are parallel or not parallel.

1. \[ y = \frac{3}{4}x + 6 \] \quad 2. \[ y = 6x \]
\[ y = \frac{3}{4}x - 2 \]

3. \[ y = \frac{1}{2}x + 1 \] \quad 4. \[ y = \frac{1}{2}x + 4 \]
\[ y = 2x - 3 \] \quad \[ y = \frac{4}{8}x - 1 \]
Course 11

Graphs and Linear Equations

\[ 2x + 3y = 6 \]
\[ y = \frac{2}{3}x - 1 \]

\[ 4x + 5y = 20 \]
\[ y = -\frac{4}{5}x + 2 \]

\[ -x + y = -y \]
\[ x - y = 6 \]

\[ y = x \]
\[ y = 7 \]

Determine which line is parallel to the line that passes through the following points. Show your work.

A. \( y = 2x + 4 \)
B. \( y = -3x - 6 \)
C. \( y = \frac{2}{3}x + 1 \)
D. \( y = -\frac{3}{4}x + 4 \)
E. \( y = 3 \)
F. \( y = 8x \)

The line that passes through these points:

9. \((0,0)\) and \((8,-6)\) is parallel to: ___

10. \((-5,1)\) and \((-4,9)\) is parallel to: ___

11. \((-4,-3)\) and \((1,-3)\) is parallel to: ___

12. \((1,-3)\) and \((-1,-7)\) is parallel to: ___

13. \((6,3)\) and \((4,9)\) is parallel to: ___

14. \((-2,2)\) and \((7,8)\) is parallel to: ___

Extension Activity:
The definition of a parallelogram is “a quadrilateral whose opposite sides are parallel.” Given a quadrilateral whose vertices are \(A (3,7)\), \(B (6,2)\) \(C (1,-3)\), and \(D (-2,2)\), determine whether it is a parallelogram or not.
Module: Perpendicular Lines and Their Slopes

Objective: To practice finding the slope of lines that are perpendicular

Name: _______________________  Date: __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

- Two lines are _______________ (parallel, perpendicular) if they intersect to form right angles.
- If the product of the slopes of two lines is -1, then the lines are perpendicular. _______________ (true, false)
- The above statement is true only if neither line is _______________ (vertical, horizontal).
- The slope of a vertical line is undefined. _______________ (true, false)
- To find the slope of a line perpendicular to another line, you must find the _______________ (positive, negative) reciprocal of the first line.

Problem Set:
Determine whether the following pairs of lines are perpendicular or not perpendicular.

1. \[ y = \frac{1}{6} x + 3 \]  
   \[ y = 6x - 2 \]

2. \[ y = -\frac{2}{3} x + 5 \]  
   \[ y = \frac{3}{2} x - 4 \]

3. \[ y = -x + 4 \]  
   \[ y = x - 3 \]

4. \[ y = \frac{5}{3} x - 6 \]  
   \[ y = -\frac{6}{10} x + 1 \]
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5. \( x + 4y = 12 \)
   \( y = 4x - 2 \)

6. \( y = \frac{2}{3}x + 5 \)
   \(-3x + 2y = 12\)

7. \( y = \frac{1}{2}x \)
   \(2x + y = 11\)

8. \( 8x - 6y = 24\)
   \(3x - 4y = 6\)

Determine whether the line passing through the given pair of points is perpendicular to any of the given lines.

A) \( y = 2x + 4 \)
B) \( y = -3x - 6 \)
C) \( y = \frac{2}{3}x + 1 \)
D) \( y = -\frac{3}{4}x + 4 \)
E) \( y = 3 \)
F) \( y = 8x \)

9. (-1,-7) and (5,-5) perpendicular to _____
10. (5,-2) and (-3,-1) perpendicular to _____

11. (-3,-4) and (3,4) perpendicular to _____
12. (0,-2) and (6,-5) perpendicular to _____

13. (3,-2) and (-7,4) perpendicular to _____
14. (2,7) and (2,-3) perpendicular to _____

Extension Activity:
The definition of a rectangle can be stated as “a quadrilateral with four right angles.” Given a quadrilateral with vertices A (-7,4), B (-4,6), C (-2,3), and D (-5,1), determine whether or not it is a rectangle.
Fill in the blanks. Use one of the words in parentheses when choices are given.

- You can identify the slopes of lines that are parallel or perpendicular to a given line by using ____________________ (equations, numbers) or ________________. (lines, graphs)

- When you know the slopes, you can determine the _______________ (relationship, distance) between the graphs of two equations.

- If the slopes of the lines of two equations are the same, the lines are ________________. (parallel, perpendicular)

- If the slopes of the lines of two equations are negative reciprocals of each other, the graphs are ________________. (parallel, perpendicular)

- When the slopes are neither the same nor are they negative reciprocals, the graphs of the equations are not parallel or perpendicular. ________________. (true, false)

Problem Set:
Determine whether the following pair of lines are parallel, perpendicular, or neither.

1. \[ y = 3x - 2 \]
   \[ y = \frac{1}{3} x + 6 \]

2. \[ y = 4x + 5 \]
   \[ y = -7 + 4x \]
### Course 11  
#### Graphs and Linear Equations

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<td>4.</td>
<td>( y = \frac{1}{2}x - 3 )</td>
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<td>(-x - 2y = 10)</td>
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<td>5.</td>
<td>( y = 4x - 9 )</td>
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<td>( 4x - y = 16 )</td>
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<td>6.</td>
<td>( 2x + 5y = 10 )</td>
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<td>( 6x + 15y = 30 )</td>
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<td>( y = \frac{1}{3}x + 2 )</td>
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<td>( 2x = 4y )</td>
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<td>( y = -2x + 3 )</td>
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<td>( y = 6 )</td>
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<td>13.</td>
<td>( x = 4 )</td>
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<td>( y = 10 )</td>
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<tr>
<td>14.</td>
<td>( 8x - 4y = 12 )</td>
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<tr>
<td></td>
<td>(-4x + 2y = -6)</td>
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</tbody>
</table>

**Reflection:**

How do you verify whether the lines of two equations are parallel or perpendicular without graphing them?

________________________________________________________________________

________________________________________________________________________

________________________________________________________________________
Fill in the blanks.

- Lines that rise to the ______________ have a negative slope. Lines that rise to the ______________ have a positive slope.
- The slope of every horizontal line is ______________. The slope of every vertical line is ______________.
- All y-intercepts have an x-coordinate of ______________.
- The slope-intercept form of an equation is ______________.

Problem Set:
Write the following equations in slope-intercept form. Then identify the slope and the y-intercept. Note which direction the graph of the equation will rise —left or right.

1. $4x + 2y - 1 = 7$  
   slope = _______  
   y-intercept = _______  
   graph rises _______

2. $y = 2x$  
   slope = _______  
   y-intercept = _______  
   graph rises _______

3. $5y - 4x = 30$  
   slope = _______  
   y-intercept = _______  
   graph rises _______

4. $4x - y + 5 = 0$  
   slope = _______  
   y-intercept = _______  
   graph rises _______

5. $-2x + y - 2 = 2$  
   slope = _______  
   y-intercept = _______  
   graph rises _______

6. $2y = 4x + 18$  
   slope = _______  
   y-intercept = _______  
   graph rises _______
Course 11  Graphs and Linear Equations

7.  \( x + 4y = -8 \)  
8.  \( 9x + 8y = 32 \)  
9.  \( 4y = 36 - 5x \)

slope = _______   
slope = _______   
slope = _______

\( y \)-intercept = _______   
\( y \)-intercept = _______   
\( y \)-intercept = _______

graph rises _______   
graph rises _______   
graph rises _______

Identify the equations above that are parallel and explain why.

Identify the equations above that are perpendicular and explain why.

Graph two of the above equations on the coordinate plane. Label the lines with their equations.

Plot and label the \( y \)-intercepts of lines A and B. Then plot an additional point on each line. Third, find the slope of each line. Last, write the equation of each line in slope-intercept form.
Fill in the blanks using these words: identical, intersecting, not intersecting.

When you solve a system of equations by graphing, there are three possible results:

- If the lines are _____________, there is one solution \((x,y)\).
- If the lines are _____________, the solution set is an empty set.
- If the lines are _____________, all numbers on the line are solutions.

Problem Set:
Solve each system by graphing the equations.

1. \[\begin{align*}
y &= \frac{1}{2}x - 4 \\
y &= -3x + 3
\end{align*}\]

solution:

2. \[\begin{align*}
y &= -2x - 6 \\
y &= -2x - 2
\end{align*}\]

solution:
Course 12  Linear Systems of Equations and Inequalities

3. \[ y = \frac{1}{4}x + 1 \]
   \[ y = 4x + 4 \]

4. \[ y = \frac{1}{3}x + 4 \]
   \[ y = -\frac{1}{3}x \]

5. \[ 5x + y = 3 \]
   \[ y = -3 + x \]

6. \[ y = -2x \]
   \[ y = x + 3 \]

Extension Activity:
Solve this system by graphing the equations. Then check your answer by making sure that the solution satisfies each equation. Show your work.

\[-2 + y = x \]
\[ 6 - 3x = y \]
\[ -7x + 4 + y = 0 \]

Reflection:
The systems of equations for problems 2 and 4 had the same solution. For each of these problems, compare the slopes of the two equations. Can you find a pattern?
Fill in the blanks using these words: slopes, y-intercepts.

It's possible to find the solution type of a system of linear equations by comparing the slopes and y-intercepts.

- If the lines are parallel, the _______ are the same.
- If the lines are coincident, the _______ and the _______ are the same.
- If the lines intersect, the _______ must be different.

Problem Set:
Complete the steps for each system of equations:
1) Rewrite the equations in slope-intercept form.
2) Classify the system as parallel, coincident, or intersecting.

1. \(5 - x = y\)
   \[y = 5x\]
   \[y - x = -7\]
   \[y = x - 7\]

2. \(y = \frac{1}{2}x + 8\)
   \[3y = x - 6\]
   \[2x + y = -3\]
   \[6x + 9 = -3y\]
Course 12  Linear Systems of Equations and Inequalities

5. \( y = \frac{1}{3} x + 2 \)  \hspace{1cm} 6. \( 3x + 3y = 6 \)
\[ \frac{1}{3} x = y \hspace{1cm} y = 3x + 2 \]

7. \( y = \frac{1}{4} x - 2 \)  \hspace{1cm} 8. \( -9x = 2 + y \)
\[ -x + 4y = -8 \hspace{1cm} 7 + y = -9x \]

9. \( 3y = 12 - x \)  \hspace{1cm} 10. \( 7 - y = x \)
\[ y = 4 - \frac{1}{3} x \hspace{1cm} 2x + y = 6 \]

**Extension Activity:**
Create a system of equations for each solution type: parallel, coincident, and intersecting.

parallel:

coincident:

intersecting:

**Reflection:**
Which of the three systems was most difficult to create?

________________________________________________________________________

Why?

________________________________________________________________________
Fill in the blanks.

- When you solve a system of inequalities by graphing, there are three possible results: _________________ solutions in common, no solutions in common, all solutions in common.
- When a _______________________ of inequalities is graphed, the inequalities are shown on one coordinate plane.
- The solution of a system of inequalities is the area where the solutions _______________________.

Problem Set:
Solve each system by graphing the inequalities. Make sure to highlight the solution area.

1. \[
\begin{align*}
y &< 3x \\
y &\geq -3x - 4
\end{align*}
\]

2. \[
\begin{align*}
y &> x - 2 \\
y &> -x + 1
\end{align*}
\]
### Course 12  Linear Systems of Equations and Inequalities

3. \[ y \leq \frac{1}{2}x + 2 \]
   \[ y > 4x - 3 \]

4. \[ y > -\frac{1}{4}x + 1 \]
   \[ -4 > -x - 4y \]

5. \[ 3 - y \leq -2x \]
   \[ y \leq 2x - 1 \]

6. \[ y < 3 + x \]
   \[ y - x \leq -3 \]

**Extension Activity:**
Notice the system of inequalities and the graph below. What pattern in the inequalities allows you to predict how the boundary lines of the inequalities are related?
Module: Solving Linear Systems of Equations: Substitution Method

Objective: To practice solving a system of equations by substitution

Name: _______________________ Date: __________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ Eliminate a variable in one equation by _________________ an expression from the other equation for the ________________.

➢ Solve the other ________________ for the value of one of the variables.

➢ Substitute the value in ________________ (either, both) equation(s).

➢ Solve the equation for the value of the ________________ (first, second) variable.

Problem Set:
Use the substitution method to solve. Show your work.

1. \[ 2x + 4y = 12 \]
   \[ x = y - 6 \]

2. \[ 3x + 2y = 46 \]
   \[ x - y = 7 \]

3. \[ 7x - 2y = 22 \]
   \[ y = x + 4 \]

4. \[ 2x + 6y = 16 \]
   \[ x + y = 14 \]
Course 12  Linear Systems of Equations and Inequalities

5.  \[3x - y = -10\]  \[4x + 4y = 40\]  
6.  \[5x + 2y = 20\]  \[3x = y - 1\]  

7.  \[x + 2y = 7\]  \[x = \frac{3}{2}y\]  
8.  \[2x + 3y = 4\]  \[y = -x\]  

9.  \[x = 3y\]  
10.  \[4y = 2x - 2\]  
\[3y - 4x + 30 = -60\]  \[y = x + 1\]  

11.  \[2x + y = -9\]  
\[x + 4y - 6 = 0\]  
12.  \[2x + 2y = 16\]  
\[x - y - 3 = 3\]  

Reflection:
Write how you would explain the substitution method to a friend. Illustrate your word explanation with an example. Use the back of this page to do this problem.

________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
________________________________________________________________________
### Fill in the blanks.

- Arrange the equations so that like terms are in the same ______________.
- You must ______________ one or both of the equations when the coefficients of one of the variables are not the same or additive inverses.
- ______________ if the coefficients are additive inverses.
- ______________ if the coefficients are the same.

### Problem Set:
**Use the addition method to solve. Show your work.**

1. \[2x + y = 3\]  
   \[-2x + 3y = -1\]

2. \[x + 6y = -14\]  
   \[-x + 4y = -6\]

3. \[3x + y = 0\]  
   \[6x - y = 18\]

4. \[2x - 3y = 9\]  
   \[-5x + 3y = 0\]

5. \[3x - y = -12\]  
   \[4x - 3y = -31\]

6. \[4x = 5y + 25\]  
   \[2x = 7y + 17\]
Course 12  
Linear Systems of Equations and Inequalities

7. \[2x + 3y = 46\]  \[3x - 4y = 1\]  
8. \[5y = 2x - 20\]  \[3y = 7x - 12\]

9. \[7y - 5x - 5 = 0\]  \[2y - 3x = 3\]  
10. \[4x + 3y = -6\]  \[5x + 4y - 7 = 0\]

11. \[-4x + 3y = 16\]  \[5x - 5y = -15\]  
12. \[2x + 7y = 21\]  \[5x + 2y = 6\]

Reflection:
Write how you would explain the addition method to a friend. Illustrate your word explanation with your own example.
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ To solve a system of linear equations using matrices, you must transform the matrix. The rules for transforming a matrix are:
  - Any two rows can be exchanged (exchanged, removed).
  - A row can be multiplied by any non-zero real number.
  - A row can be added to another row or to a multiple of another row.

➢ To solve a system of linear equations using matrices, you must transform the matrix into this format, where \( p \) and \( q \) are constants, variables, or letters:

\[
\begin{bmatrix}
1 & 0 & | & p \\
0 & 1 & | & q
\end{bmatrix}
\]

Here’s how:
  - Start with the first column. Transform the element into a number.
  - Transform all the other elements in the column into zeros.
  - Start on the second column. Transform the element into a number.
  - Transform all the other elements in the column into zeros.

Problem Set:
Write the augmented matrix that represents the systems of equations.

1. \(4x + y = 15\) \hspace{1cm} 2. \(y = x + 6\)
\[\begin{align*}
3x + 6y &= 10 \\
2x - 7y &= 20
\end{align*}\]
Course 12  Linear Systems of Equations and Inequalities

3. \[12x = 42\]  \[y - 3x = 12\]  

4. \[-y + 21 = 4x\]  \[6y = 18\]

Solve the system of equations by setting up an augmented matrix and transforming it into the format \[
\begin{bmatrix}
1 & 0 & | & p \\
0 & 1 & | & q
\end{bmatrix}
\]

where \(p\) and \(q\) are constants.

5. \[2x + 8y = 36\]  \[x + 5y = 10\]

6. \[x - y = 1\]  \[5x = 12\]

7. \[-3x - 4y = -7\]  \[2x + 7y = -4\]

8. \[-2x + y = -5\]  \[x - 8y = 40\]

Extension Activity:
Solve this system of equations using a matrix. Then graph the solution.

\[
\begin{align*}
3x + 2y &= 19 \\
-12x + 8y &= -44
\end{align*}
\]

solution:

Reflection:
How is solving a system of equations with matrices similar to solving a system of equations by adding?
Answers will vary, but should include the idea that both methods involve transforming equations so the terms will zero out.
Fill in the blanks.

➢ To solve a system of three ____________ equations such as
  
  \[ x + 3y - 2z = 2, \quad 4x - 7y = 5, \quad 3y - 2 = 12, \]
  
  first set up the system in matrix form. Put the _____________ of the variables in the first three columns
  and the constants in the _____________ column. Include the minus sign
  for _____________ coefficients. Remember, \( x = 1x \) and \( 0x = \) __________.

➢ Next, _____________ the matrix so that it takes this form.

\[
\begin{bmatrix}
1 & 0 & 0 & | & p \\
0 & 1 & 0 & | & q \\
_ & _ & _ & | & r
\end{bmatrix}
\]

➢ Since the _____________ in the matrix represent constants and
  coefficients, the transformation will give you:

  \[ x = p \]
  \[ y = q \]
  \[ _ = r \]

Problem Set:
Write the augmented matrix that represents the system of equations.

1. \( 3x + 2y = z \)
   \[ 4x + y - z = 16 \]
   \[ x + y + 2z = -8 \]
Course 12  Linear Systems of Equations and Inequalities

2. \(2x - z = y + 3\)
   \(-x + 3y = 6\)
   \(3z - x + 4 = y\)

3. \(x + y + z = 12\)
   \(4x - y - z = 1\)
   \(-x + 2y + 3z = 6\)

4. \(12x - 5y + z = 32\)
   \(8y - 5x + 10z = 50\)
   \(z + x - 2y = 18\)

Solve the system of equations by setting up an augmented matrix and transforming it into the format where \(p\), \(q\), and \(r\) are constants.

\[
\begin{bmatrix}
1 & 0 & 0 & p \\
0 & 1 & 0 & q \\
0 & 0 & 1 & r
\end{bmatrix}
\]

5. \(x + y + z = 12\)
   \(3x + 2y - z = 4\)
   \(-2x - 4y + 3z = 3\)

6. \(z + x = 5\)
   \(-7x + y = -4\)
   \(5y - 3z = 41\)

7. \(x - 2y = 44 - 4z\)
   \(3x + y - 11 = 17\)
   \(x - y = 4\)

8. \(-2x + y - z = 10\)
   \(4x - 5y + 6z = 41\)
   \(2y - 4z = -26\)

Reflection:
You know how to solve a system of two equations by setting up a matrix and transforming it into this format:

\[
\begin{bmatrix}
1 & 0 & p \\
0 & 1 & z
\end{bmatrix}
\]

You know how to solve a system of three equations by setting up a matrix and transforming it into this format:

\[
\begin{bmatrix}
1 & 0 & 0 & p \\
0 & 1 & 0 & q \\
0 & 0 & 1 & r
\end{bmatrix}
\]

Think about the patterns in these matrices. How would you explain to a friend the process for solving a system of four equations and four variables by setting up a matrix and transforming it?
Course 12  Linear Systems of Equations and Inequalities

Module: Solving Problems with Linear Systems

Objective: To practice solving word problems using a system of two linear equations or inequalities

Name: _______________________  Date: __________________

Fill in the blanks. Then number the steps in the correct order.

➢  Use the information from the _____________ to write a system of _____________ or _____________. (equations, problem, inequalities)

➢  Read the _____________ carefully. Decide what questions you need to _____________. (question, answer, problem, know)

➢  Use one of the methods you know _____________, ____________, ____________, _____________, to solve the problem. (graphing, addition, subtraction, substitution, matrices, multiplication)

➢  Assign _____________ to the unknowns. (values, variables)

Problem Set:
Use the steps for solving a system of linear equations or inequalities to answer the questions.

1. Sam paddles his canoe upstream, against the current, for 12 miles. It takes him 2 hours. Coming back downstream, with the current, he covers the same distance in only an hour. How fast can Sam paddle his canoe in still water? How fast is the current?

2. A restaurant manager needs to order at least 12 large cans of tomato sauce and at least 40 bags of flour. The tomato sauce costs $3 per can and the flour costs $8 per bag. The restaurant manager has $400 to spend. If \( x \) = cans of tomato sauce and \( y \) = bags of flour, which of these sets of equations describes the situation?
   a. \( x \leq 12 \quad y \leq 40 \quad 8x + 3y \leq 400 \)
   b. \( x \geq 3 \quad y \geq 8 \quad 12x + 40y \leq 400 \)
   c. \( x \geq 12 \quad y \geq 40 \quad 3x + 8y \leq 400 \)
3. If the restaurant manager in the above problem buys 20 cans of tomato sauce, can she still buy enough flour? Why?

4. The candy store sells peppermints and truffles in small, single-item boxes. There are 100 boxes, 65 plain brown cardboard boxes and 35 red gift boxes. Only truffles belong in the red boxes, but either peppermints or truffles can go in the plain boxes. Which system of inequalities best represents the situation? Let $x$ represent truffles and $y$ represent peppermints.

   a. $x \leq 35$  
      \[ x + y \leq 100 \]
   b. $y - x \leq 35$  
      \[ x + y \leq 100 \]
   c. $x \leq 100$  
      \[ x + y \leq 65 \]

5. Using the same candy store problem, which combination of truffles and peppermints would fit into the boxes?

   a. 52 truffles, 48 peppermints
   b. 20 truffles, 85 peppermints
   c. 26 truffles, 74 peppermints

**Extension Activity:**

**How can you check your answers, when solving linear systems of equations and inequalities?** Take the values you find for $x$ and $y$ and try them in your original equations.

Here’s how one student solved a problem for a boat traveling 30 miles with the current in 6 hours and 30 miles against the current in 10 hours.

\[ x = \text{speed without current} \quad y = \text{speed of the current} \]

\[ x + y = \text{rate with the current} \]

\[ x - y = \text{rate against a current} \]

\[ 30 = (x + y) \cdot 6 \quad 30 = (x - y) \cdot 10 \]

\[ 5 = x + y \quad 3 = x - y \]

\[ 5 = x + y \]

\[ 3 = x - y \]

\[ 8 = 2x \]

\[ 4 = x \]

\[ 1 = y \]

Now, try putting $x = 4$ and $y = 1$ back into the original equations. Do the values check?

\[ 30 = (x + y) \cdot 6 \quad 30 = (x - y) \cdot 10 \]

\[ 30 = \left(\_\_+\_\_\right) \cdot 6 \quad 30 = \left(\_\_--\_\_\right) \cdot 10 \]
Module:  Review:  Linear Systems

Objective:  To practice finding the solution sets of linear equations and inequalities

Name:  _______________________  Date:  ________________

Fill in the blanks.

➢ To solve a system of inequalities, graph the two inequalities and identify where the solutions ________________. That is the ________________ of the system.

➢ To solve a system of equations using the ________________ method, solve one of the equations for one of the variables. Substitute the expression for the variable in the other equation and solve. Then substitute the value for the variable in either equation and solve for the second variable.

➢ To solve a system of equations using the ________________ method, look to see whether the coefficients of either variable are the same or additive inverses.

Problem Set:

Use the graphing method to solve the system of inequalities. Show your work in the space provided.

\[ y < 2x - 5 \]
\[ y + 6 \geq -3x \]
Course 12  Linear Systems of Equations and Inequalities

Solve the system of equations using the substitution method, the addition method, and a matrix. Show your work in the space provided.

\[
\begin{align*}
  x - y &= 1 \\
  4x &= 5y + 2
\end{align*}
\]

**Substitution Method**

**Addition Method**

**Matrix**

Reflection:
Think about the methods you used to solve the system. Which method worked best for this system? Why?
________________________________________________________________
________________________________________________________________

Which types of problems are best solved using the substitution method? Which types are best solved using the addition method?
________________________________________________________________
________________________________________________________________
Module: Chance Experiments and Probability

Objective: To practice describing the sample space, the event space, and the probability of a chance experiment

Name: _______________________  Date: ________________

Fill in the blanks. Use one of the words in parentheses when choices are given.

➤ The set of all possible outcomes of a chance experiment is the ______________ (sample, event) space of the experiment.

➤ The description of the elements of an event is called the ______________ (sample, event) space

➤ Complete the formula for probability:

   Probability of an event = number of elements in a(n) ______________ number of elements in a(n)

➤ Suppose you toss a coin twice. In this case, one toss turns up heads and the other toss turns up tails. Describe this outcome by writing it as an ordered pair. ______________

➤ A measure of how likely it is that an event will occur is called ______________ (chance, probability).

Problem Set:
Identify whether a problem describes a sample space or an event space.

➤ A bag of candy has 4 red, 6 green, and 7 blue candies ______________

➤ You pick one red candy out of the bag ______________
Course 13

Probability

Find the probability of the event described in the following situations. Show your work. Write the answer in simplified form.

Gillian works at a day care. There are 12 kids in her group. Today, 4 want to play tag, 2 want to play catch, and 6 want to play on the playground equipment. Gillian puts each request in a hat and asks one of the children to draw an activity. What is the probability that the game of catch will be the activity she chooses?

Ming stocks shelves at the grocery store. Today, he puts 16 cans of tomato soup, 20 cans of chicken noodle soup, and 24 cans of vegetable soup on a shelf. What is the probability that a can of tomato soup will be the first one chosen?

Umberto has invited a total of 15 people to his birthday party: 6 cousins and 9 classmates. What is the probability that a cousin will be the first to arrive?

There are 14 boys at a party: 4 boys are excellent dancers, 3 are good dancers, and 7 don’t dance at all. Nichole doesn’t know the boys’ skills, but she decides to ask one of them to dance anyway. What is the probability that she will choose an excellent dancer to be her partner (without first watching them dance)?

Savin has 36 coins in her coin purse. She has 18 pennies, 9 quarters, 4 dimes, and 5 nickels. She needs a quarter for the parking meter. If she doesn’t look, what is the probability that she will pull out a quarter on the first try?

Reflection:
Write a probability problem from your own experience.

________________________________________________________________
________________________________________________________________
________________________________________________________________
Fill in the blanks. Use one of the words in parentheses when choices are given.

- An ________________ (experiment, occurrence) is an event that has more than one possible outcome.
- The probability that an event or its complement will occur equals ______.
- The probability of any given event in an experiment is the ________________ (ratio, likelihood) of the number of ways the event can occur to the number of ________________ (different, possible) outcomes.
- When the probability of an event is equal to 1, the event is ________________ (likely, certain) to occur. When the probability of an event is equal to 0, the event is ________________ (impossible, nonexistent).

Problem Set:
Answer the following questions on probability.

A die has 6 possible outcomes: {1,2,3,4,5,6}. What is the probability that you will roll a number greater than 6?

Marshall has a bag of 12 red gumballs. What is the probability that he will choose a red gumball from the bag?

In Jenell’s class, there are 14 girls and 20 boys. Each name is put into a hat for a prize. What is the probability that a boy’s name will not be drawn?
In the same class, what is the probability that a boy or girl’s name will be drawn?

Tara is going to make a birthday cake for her friend Kenna. She has 5 choices — chocolate, white, lemon, carrot, and spice. Assuming that Tara and Kenna like all the choices equally and that Tara closes her eyes to choose, what is the probability that Tara will pick either carrot or lemon?

What is the probability that she will not pick carrot or lemon?

Shaylee and Tanner are playing a board game that has 84 squares. Of the total 24 squares have penalties. What is the probability that Shaylee will not land on at least one penalty square during the game?

Marcos wants to take a canoe trip in the Boundary Waters Canoe Area. He has time for a 7-day trip and he has narrowed his choices to 12 possible routes. Of the total, 3 routes are supposed to have the best fishing in the BWCA, 4 have pictographs (rock drawings) along the way, and 5 have the best chances of seeing a moose. If he were to write down each of his choices on a slip of paper, put them in a hat, and randomly choose 1 route, what is the probability that he will choose a route that has pictographs?

What is the probability that he will choose a route that does not have pictographs?

Reflection:
Suppose you’re the teacher. Explain to the class why the probability of event and its complement add up to 1.
Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ To determine the total number of possible outcomes in an event, you need to ________________ (add, multiply).

➢ When one event does not influence the outcome of the other events, it is said to be ________________ (independent, dependent).

➢ If an event A has \( m \) possible outcomes and event B has \( n \) possible outcomes, then there are ________________ \((mn, AB)\) different ways for events A and B to occur together.

Problem Set:
Solve the problems. Show your work.

There are 16 basketball teams in the tournament. The 8 high-ranking teams are paired randomly with the 8 low-ranking teams for the first round. How many combinations of teams are possible for the first round?

In Garrett’s state, license plates have letters in the first four spaces and numbers, 0–9, in the last two spaces. How many combinations of letters and numbers are there for license plates in her state?

Sophie is buying new hiking boots. She has 7 styles to choose from in her size. Each style has 5 kinds of laces and 4 colors. How many choices of hiking boots does Sophie have?
Course 13

Probability

Jamal is buying an ice cream sundae. He can choose from 23 kinds of ice cream, 8 different sauces, and 6 kinds of sprinkles. If he chooses to order just one kind of ice cream, one sauce, and one kind of sprinkles, how many different sundaes could Jamal order?

Esmeralda is making posters for the school art fair. She has 3 sizes of poster board. Each size comes in 5 colors. She has 6 different colors of marking pens to choose from. Assuming that she wants to make each poster different from the others by using 1 color marker, 1 color poster board, and 1 size, how many posters could she make?

Alejandro is refinishing an old desk for his room. He has stripped off all the old paint and wants to stain it, varnish it, and put on new handles. At the store he tries to decide among 3 kinds of stain, 2 kinds of varnish, and 6 styles of handles. How many different looks could Alejandro’s desk possibly have?

Extension Activity:
Solve the problems. Show your work.

Angelo works at a sandwich shop. Customers can choose from 5 kinds of meat, 3 kinds of cheese, and 4 kinds of dressings. They have a choice of whole wheat, white, and cheese bread. In addition, they can choose whether or not to have tomatoes, peppers, lettuce, or onions. How many sandwich combinations can Angelo possibly make?

Carla works in a costume jewelry store. There are 25 styles of earrings that come in 4 colors, 20 styles that come in just silver or gold, and 15 styles that come in 10 colors. In addition, all the styles can be bought as pierced or clip-on. How many different earrings does the jewelry store sell?
Fill in the blanks. Use one of the words in parentheses when choices are given.

➢ In a chance experiment, probability measures the ____________
   (likelihood, chance) a particular event will occur.

➢ The _____________ (sample, event) space is the set of all possible outcomes of a given experiment.

➢ The probability of any given event in an experiment is the ____________
   (ratio, likelihood) of the number of ways the event can occur to the number of all possible outcomes.

➢ When the probability of an event is equal to ________________ the event is certain to occur. When the probability of an event is equal to ________________ the event is impossible.

➢ It is possible to find the probability of an event not occurring. ____________ (true, false)

➢ The sum of the probabilities of an event and its complement is ____________.

➢ Complete the formula for probability:

\[
\text{Probability of an event} = \frac{\text{number of elements in a(n)}}{\text{number of elements in a(n)}}
\]
Course 13

Problem Set:
Solve the problems. Show your work.

Ben and Amy are going to play a board game. There are 7 playing pieces to choose from: a monkey, a parrot, an ostrich, a rhinoceros, a lion, a giraffe, and an elephant. If he doesn’t look, what is the probability that Ben will choose the lion?

What is the probability that he will not choose the lion or the giraffe?

What is the probability that he will choose a bear?

Jasmine works in a women's shoe store. The store sells 14 brands of shoes. There are 10 brands that come in 10 sizes, and 4 brands that come in 12 sizes. All these shoes come in black as well as other colors. How many black pairs of shoes does the store carry?

Reflection:
Explain the multiplication principle to a friend. Use examples if you want to (but try making up your own)!

________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
________________________________________________________________
Module: Introduction to Vectors

Objective: To practice how to represent situations and problems using vectors

Name: _______________________  Date: ________________

Fill in the blanks.

- ______________ coordinates are represented using \((r, \theta)\).
- ______________ coordinates are represented using \((x, y)\).
- To translate between rectangular and polar coordinates, you can use the ____________ \_______________________.
- The ______________ of a vector is represented by the length of the straight line from tail to tip.
- The ______________ of a vector is represented by the angle between the vector and some reference axis.
- The displacement of an object is known as long as its ______________ position and ______________ position are known.

Problem Set:
Identify the situation as either a scalar or a vector. Write scalar or vector in the blank provided.

1. northwest at 49.1 meters per second __________
2. 7 bananas __________
3. 29° Celsius in Memphis __________
4. beam sags when 1800-Newton force presses down on it __________
5. 88 kilometers per hour, east __________
6. 1.7 liters of motor oil __________
7. 112 kilometers north from Bay City to Delafield __________
Course 14  **Vectors**

Using a ruler, find the magnitude of each vector in millimeters.

8. _______ mm  9. _______ mm  10. _______ mm

Assume that 1 mm represents 5 Newtons [N] of force. Calculate the magnitude of force that each of the above vectors represents. Write the magnitude in the blank below.

11. _______ N  12. _______ N  13. _______ N

14. Are any of the three vectors equivalent?  **yes**  **no**

**Extension Activity:**

Draw each vector on the coordinate plane to match the situation. Label the vector with its corresponding letter. Vectors don’t need to start at (0,0).

A. An automobile is driven west at 40 kilometers per hour.

B. A runner sprints 15 kilometers per hour, south.

C. A cyclist coasts northeast at 28.3 kilometers per hour.
   (Hint:  \( c^2 = a^2 + b^2 \) where \( a = b \))

D. A fishing boat travels northwest at 42.4 kilometers per hour.
   (Hint:  \( c^2 = a^2 + b^2 \) where \( a = b \))
Course 14  Vectors

Module: Vector Addition

Objective: To practice adding vectors to solve problems

Name: _______________________  Date: __________________

Fill in the blanks.

➢ A ______________________ vector is the sum of two or more vectors.

➢ When using the tail-to-tip method to add vectors geometrically, place the

___________ of one vector on the tip of another.

➢ Another geometric method to add vectors uses the __________________

law. This method is based on properties of a polygon with four sides.

Write T if the statement is true or F if the statement is false.

➢ Vectors with like units can be added in any order. ______

➢ The algebraic method of vector addition works only if the tail of the first

vector starts at (0,0). _____

➢ For a moving object, the magnitude of distance must always equal the

magnitude of displacement. _____

Problem Set:

1. The tug-of-war competition is underway at the state fair. The orange team is
pulling the rope east with 220 Newtons of force. The purple team is pulling
the rope west with 160 Newtons of force. If the center of the rope is assumed
to be at (0,0), find the magnitude and direction of the resultant force on the
rope. (Hint: The Pythagorean theorem is not needed for this problem.)

    magnitude: __________ [Newtons]  direction: __________

2. Based on the direction of the resultant vector, is the orange team or the
purple team winning the tug-of-war?  orange team  purple team
Course 14  Vectors

Use the following sketch for questions 3, 4, 5, and 6. Write your answers on a separate sheet of paper.

3. A machine is programmed to cut a pattern in a circular piece of steel. The blade starts at the center (0,0) and moves along the path shown. Find the coordinates \((x,y)\) for the final blade position if (0,0) is considered the origin.

4. An engineer is responsible for ordering the steel piece. The steel supplier can only offer the engineer a radius from this list: 40 mm, 45 mm, 50 mm, 55 mm, 60 mm, 65 mm, or 70 mm. Given this information, what radius should the engineer order so she doesn’t come up short on material but also wastes as little as possible?

5. If a blade becomes dull after cutting a total distance of 71,000 mm, how many patterns (pieces) can the engineer expect to cut with 1 blade?

6. The blade represents a ________ vector. (displacement, force, velocity)

Extension Activity:
On a separate sheet of paper, add the three vectors geometrically. When complete, circle the letter of the correct resultant vector.